

Wavy Horizons?

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Abstract

We describe the application of a gravity wave-generating technique to certain higher dimensional black holes. We find that the induced waves generically destroy the event horizon producing parallelly propagated curvature singularities.

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1 INTRODUCTION

Interest in black hole uniqueness theorems began some thirty years ago with the pioneering work of Israel[1]. The no-hair results are now rigorously established for Einstein gravity coupled to Maxwell fields and various other simple matter systems[2]. More recently, physicists have become interested in theories with more complicated matter field couplings, as well as spacetime dimensions beyond four. While this research has produced a plethora of new solutions[3], they are found to respect the spirit of the no-hair theorems in that the black hole geometries are still completely determined by some small set of charges.

An interesting corollary of the early theoretical investigations of black holes was that each connected component of a stationary horizon must have the topology of a two-sphere[4]. One might regard this result as indicating black holes carry no ‘topological hair.’ This result is easily evaded, however, in higher dimensions. As a simple example, consider the four-dimensional Schwarzschild metric combined with a flat metric on R^n . This space-time is an extended black hole solution of Einstein’s equations in $4+n$ dimensions, and the topology of the horizon is $S^2 \times R^n$. Clearly, this straightforward construction is easily extended to constructing many other higher-dimensional black holes whose horizons inherit the topology of the ‘appended’ manifold.¹

While higher-dimensional black holes might have ‘topological hair’, one still expects that the spirit of the no-hair theorems should be obeyed in these cases. So having fixed the horizon topology and a limited set of charges, the black hole solution should be completely determined. Our present work focuses on a potential violation of these expectations. For certain classes of solutions, it is possible to generate an infinite variety of new solutions by adding wave-like perturbations[6]. These techniques may be applied to certain extended black objects, and would apparently yield black holes with an infinite variety of wavy hair. The paper is organized as follows: Section 2 describes the wave-generating technique, and provides a simple example of a wavy black hole. Section 3 investigates the curvatures of these wavy ‘horizons’ and section 4 provides a brief discussion of our results. While the present discussion is self-contained, it is lacking in many details which the interested reader may find in ref. [7].

¹Similar solutions arise for four dimensions in the presence of a negative cosmological constant[5].

2 WAVES

We begin with a brief review of the wave-generating technique of Garfinkle and Vachaspati[6]. This method was originally developed in the context of the Yang-Mills-Higgs system coupled to gravity. However, it is straightforward to extend the construction to general supergravity (or low-energy string) theories in arbitrary dimensions[7]. The starting point for this construction is a solution with a vector field k^μ which is null, hypersurface orthogonal and Killing, *i.e.*,

$$k^\mu k_\mu = 0 , \quad \nabla_{[\mu} k_{\nu]} = k_{[\mu} \nabla_{\nu]} S , \quad \nabla_{(\mu} k_{\nu)} = 0 . \quad (1)$$

If the solution contains non-trivial matter fields, their Lie-derivative must also vanish — and certain transversality constraints[7] must be satisfied as well — in order that k^μ yields an invariance of the full solution. One then defines a new metric[6]

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + e^S \Psi k_\mu k_\nu, \quad (2)$$

while leaving all of the matter fields unchanged. The new metric $\tilde{g}_{\mu\nu}$ will also be a solution provided that the function Ψ satisfies appropriate constraints.

The first constraint is that k^μ has a vanishing Lie-derivative on Ψ . One may verify that this also ensures that the hypersurface orthogonal and Killing conditions are still satisfied with the new metric (and with the same S).² It is also obvious that k^μ remains null with $\tilde{g}_{\mu\nu}$. The remaining restrictions arise to guarantee that after the metric is shifted, the explicit form of the equations of motion remains unchanged. One finds that the matter field equations are automatically invariant. However, invariance of Einstein's equations requires that the d'Alembertian acting on Ψ vanish. Therefore given a solution with a vector k^μ satisfying eq. (1), eq. (2) yields a new solution provided

$$k^\mu \partial_\mu \Psi = 0 \quad \text{and} \quad \nabla^2 \Psi = 0 . \quad (3)$$

These perturbations can be interpreted as gravity waves as follows: Consider a coordinate system adapted to the flow of k^μ . As well as the cyclic coordinate v , there is a coordinate u given ‘roughly’ by the integral of the dual one-form $k = k_\mu dx^\mu$. As none of the fields depend on the Killing coordinate v , the only ‘time’ dependence can arise through this null coordinate u . Therefore, the perturbations are moving through the space-time at the speed of light along the u direction.

²Note it follows directly for eq. (1) that k^μ has a vanishing Lie-derivative on S .

While eq. (1) is very restrictive, this solution-generating technique has found a wide-range of applications[6, 8, 9, 10]. We will present a particular black hole which arises in low-energy superstring theory satisfying these symmetry restrictions (1):

$$ds^2 = -\left(1 - \frac{Q}{R}\right)^2 dt^2 + \frac{dR^2}{\left(1 - \frac{Q}{R}\right)^2} + R^2(d\theta^2 + \sin^2 \theta d\phi^2) + \left(dy - \frac{Q}{R}dt\right)^2 + \sum_{i=5}^9 (dx^i)^2. \quad (4)$$

Within ten-dimensional superstring theory, the complete solution includes various matter fields whose details are inessential to the following discussion. Amongst the higher dimensions, y is distinguished by the nonvanishing G_{yt} component, which indicates that momentum is flowing in this particular direction. The reader may recognize the first four terms in the line element as describing the geometry of the extremal Reissner-Nordstrom black hole. This is the four-dimensional space-time which would be observed by low-energy physicists. The full solution then describes a black hole with a horizon at $R = Q$ which has the topology of $S^2 \times R^6$.

As is evident from eq. (4), this solution has a number of Killing vectors and it is straightforward to show that the combination

$$k^\mu \partial_\mu \equiv \partial_v = \partial_t + \partial_y \quad (5)$$

is everywhere null. In fact, one finds that k^μ is the null generator of the horizon. The ∂_y contribution then indicates linear motion of the horizon along the y direction. From

$$k_\mu dx^\mu = \left(1 - \frac{Q}{R}\right) (dy - dt) \quad (6)$$

one sees that the null Killing vector k^μ is also hypersurface-orthogonal, with $e^{-S} = 1 - Q/R$. Hence the metric (4) admits the symmetry (1) required to apply the wave-generating technique.

Using eq. (2), one constructs a new metric

$$\widetilde{ds}^2 = ds^2 + e^{-S} \Psi (dy - dt)^2 \quad (7)$$

where Ψ must satisfy the constraints in eq. (3). The first condition $(\partial_t + \partial_y)\Psi = 0$ requires $\Psi = \Psi(u=t-y, r, \theta, \phi, x^i)$. Before examining the second constraint, it is convenient to shift the radial coordinate to $r = R - Q$, which transforms eq. (4) to

$$ds^2 = -f^{-2}dt^2 + f^2 \left(dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)\right) + \left(dy - \frac{Q}{fr}dt\right)^2 + \sum_{i=5}^9 (dx^i)^2 \quad (8)$$

with $f \equiv 1 + Q/r$. With this coordinate shift, the second condition reduces to

$$\left[\nabla_F^2 + f^2 \sum_{i=5}^9 (\partial_i)^2 \right] \Psi = 0 \quad (9)$$

where ∇_F^2 is the Laplacian on the *flat* space covered by (r, θ, ϕ) . The Killing constraint has ensured that no t or y derivatives appear in eq. (9).

For simplicity, we begin by considering solutions of eq. (9) which are independent of the internal coordinates x^i . In this case, the general solution is

$$\Psi = \sum_{l,m} \left(a_{lm}(u) r^l + b_{lm}(u) r^{-(l+1)} \right) P_l^m(\cos \theta) \cos(m\phi + \delta_m(u)) \quad (10)$$

where $u = t - y$, as above, and the $P_l^m(\cos \theta)$ are associated Legendre functions. In general, the phases δ_m , as well as the amplitudes a_{lm} and b_{lm} , are *arbitrary* functions of u . Now there are two classes of solutions, those which grow at large r and those which decay. In the former case with r^l and $l > 1$, the metric is not asymptotically flat³, and so these perturbations are not intrinsic to the black hole, rather they fill the asymptotic region with gravitational radiation. Hence I will focus on the decaying solutions with $r^{-(l+1)}$ profiles. These perturbations are localized near the ‘horizon’ at $r = 0$, and hence are candidates for ‘wavy’ hair on the black hole.

Generalizing to solutions of the full equation (9), one finds

$$\Psi = \sum_{l,m,n_i} b_{lmn_i} P_l^m(\cos \theta) \cos(m\phi + \delta_m) \prod_{i=5}^9 \cos(n_i x^i / R_i + \delta_{n_i}) F_{ln_i}(r) \quad (11)$$

where the x^i coordinates are assumed to be compactified with period $2\pi R_i$. Again, the phases δ_m and δ_{n_i} and the amplitudes b_{lmn_i} can have an arbitrary dependence on u . The radial functions F_{ln_i} satisfy

$$\left[\frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{l(l+1)}{r^2} - \left(1 + \frac{Q}{r} \right)^2 M^2 \right] F(r) = 0 \quad (12)$$

where $M^2 = \sum_{i=5}^9 (n_i / R_i)^2$. The solutions can be written in terms of confluent hypergeometric functions, but only a qualitative description of the solutions will be needed here. In general, there are again two classes of solutions: growing and localized. As before, the growing perturbations will be discarded as they fill the entire spacetime with gravitational waves. The localized solutions are more interesting, as they are candidates for wavy hair.

³In fact, the r^l perturbation with $l = 0$ yields a diffeomorphism of the original metric, while with $l = 1$ the solution is asymptotically flat and the perturbation produces transverse oscillations of the horizon [9, 11].

Their long range behavior is $F \sim \exp(-Mr)/r$, and hence any internal oscillations result in perturbations which decay faster than any of those in eq. (10). In the limit $r \rightarrow 0$, the solutions admit a series representation of the form: $F = r^{-\beta}(1 + \sum_{n=1}^{\infty} F_n r^n)$. From eq. (12), one finds that the leading power is

$$\beta = (1 + \sqrt{1 + 4l(l+1) + 4M^2Q^2})/2. \quad (13)$$

Hence one finds that all of the candidates for wavy hair have singular behavior at the null surface $r = 0$.

3 SINGULARITIES

The above construction appears to have provided the black hole (8) with an infinite variety of wavy hair. Such a result would certainly run contrary to the idea that higher dimensional black holes should have no hair. However, the radial profile of these waves diverges as $r^{-\beta}$ near the ‘horizon,’ and hence one must be careful to investigate whether any curvature singularities are produced at the surface $r = 0$.

A natural approach to investigate this question is to examine various curvature scalars, *e.g.*, $R_{\mu\nu}R^{\mu\nu}$ or $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$, for the existence of singularities. Upon performing the lengthy calculations to construct these scalars, one finds no evidence of a singularity at $r = 0$. In fact, one finds no evidence of the wave perturbation at all! The latter is true for all curvature scalars, which was proven with the following theorem[7]:

If $g_{\mu\nu}$ is a pseudo-Riemannian metric admitting a null, hypersurface-orthogonal, Killing vector k^μ and $\tilde{g}_{\mu\nu} = g_{\mu\nu} + \kappa k_\mu k_\nu$, where κ is any scalar Lie-derived by k^μ to zero, i.e., $\mathcal{L}_k \kappa = 0$, then all of the scalar curvature invariants of $\tilde{g}_{\mu\nu}$ are exactly identical to the corresponding curvature invariants of $g_{\mu\nu}$.

This result is purely geometric in nature, and holds for any metric satisfying the symmetry conditions in eq. (1). The transformation (2) provides a specific example where the theorem applies with $\kappa = e^S \Psi$. Hence one has the rather surprising result that all scalar curvature invariants are identical for both the original and the shifted metrics in eq. (2). Therefore these invariants do not contain any information about how the space-time geometry is modified by the wavy perturbations. Tidal forces prove to be a better probe of the wave geometry. These forces are determined by the Riemann curvature measured in the rest frame of a geodesic observer.

The tidal forces will be calculated for each mode of oscillation individually. Hence consider

as a candidate perturbation:

$$\Psi = B(u)F(r)P_l^m(\cos \theta) \cos(m\phi + \delta_m) \prod_{i=5}^9 \cos(n_i x^i / R^i + \delta_{n_i}) . \quad (14)$$

The calculation proceeds in several steps: First, one must show there exists a geodesic stretching between asymptotic infinity and the null surface — hence showing that $r = 0$ belongs to the space-time. Next, a convenient orthonormal frame is constructed. Examining the curvature in this stationary frame, one finds that all of the components are finite. One then constructs the Lorentz transformation relating the stationary frame to the rest frame of an observer moving along the geodesics identified above. Finally, the curvature is boosted to the infalling frame in order to determine the observer's tidal forces. The divergences identified in this way are equivalent to parallelly propagated curvature singularities.

The first step of identifying geodesics in the presence of a general oscillation proves to be a daunting task. To simplify the problem, we consider the case where the amplitude B and the phases δ are constants. These solutions will be enough to identify the leading divergences. The simplest approach is to choose values of θ , ϕ and x^i such that the derivatives of Ψ with respect to these coordinates vanish. Then the geodesic equations are consistently solved with these fixed values and the motion reduces to

$$\begin{aligned} dt/d\tau &= f [\omega + (Q/r + B_0 F(r)) (\omega - p)] \\ dy/d\tau &= f [p + (Q/r + B_0 F(r)) (\omega - p)] \\ dr/d\tau &= -f^{-1} [f^2 H^2 (\omega - p)^2 + 2f(\omega - p)p - 1]^{1/2} \end{aligned} \quad (15)$$

where ω and p are integration constants, $f \equiv 1 + Q/r$ and $H^2 \equiv 1 + B_0 F(r)/f$. We have also set $\Psi = B_0 F(r)$ along the geodesic at fixed values of θ , ϕ and x^i . Now in order that the geodesic reaches $r = 0$, the fixed coordinates must be chosen so that $B_0 > 0$. Also $\omega^2 > p^2 + 1$ ensures that the geodesic extends back to $r \rightarrow \infty$.

As an intermediate step, we define a stationary orthonormal basis of one-forms

$$\begin{aligned} e^t &= dt/fH & e^r &= f dr & e^y &= H(dy - dt) + dt/fH \\ e^\theta &= (r + Q) d\theta & e^\phi &= (r + Q) \sin \theta d\phi & e^i &= dx^i . \end{aligned} \quad (16)$$

One can readily verify that $\widetilde{ds}^2 = \eta_{ab} e^a e^b$ reproduces the line element in eq. (8). Calculating the curvature in this frame, one finds that all components are everywhere finite. We are particularly interested in the limit $r \rightarrow 0$ along the geodesics identified above. There the

curvature components reduce to

$$\begin{aligned}
R^{trtr} &\simeq \frac{1 - 2\beta(\beta - 1)}{4Q^2} & R^{yryr} &\simeq -\frac{1 + 2\beta(\beta - 1)}{4Q^2} & R^{tryr} &\simeq -\frac{\beta(\beta - 1)}{2Q^2} \\
R^{tatb} &\simeq \delta^{ab} l(l + 1)/4Q^2 \simeq R^{yayb} \simeq R^{tayb} & & \text{for } a, b = \theta, \phi \\
R^{titj} &\simeq \delta^{ij} n_i^2/2R_i^2 \simeq R^{yiyj} \simeq R^{tiyj} & & \text{for } i, j = 5, \dots, 9 \\
R^{tyty} &\simeq 1/4Q^2 & R^{\theta\phi\theta\phi} &\simeq 1/Q^2.
\end{aligned} \tag{17}$$

Referred to this frame, the proper ten-velocity is $V^a = e^a_\mu dx^\mu/d\tau$:

$$\begin{aligned}
V^t &= Hf(\omega - p) + p/H & V^y &= p/H \\
V^r &= -\left[f^2 H^2(\omega - p)^2 + 2f(\omega - p)p - 1\right]^{1/2}
\end{aligned} \tag{18}$$

along with $V^\theta = 0 = V^\phi = V^i$. As a check, one may easily verify that $\eta_{ab}V^aV^b = -1$.

Now we need a Lorentz transformation which takes the unit time-like vector $N^a = \delta_t^a$ into the observer's ten-velocity: $V^a = L^a_b N^b$. Applying this transformation to our stationary vielbein (16) produces a natural basis of orthonormal one-forms which the infalling observer might use in her rest frame. The simplest choice is

$$L^a_b = \begin{pmatrix} V^t & V^y & V^r & 0 \\ V^y & 1 + \frac{(V^y)^2}{V^t+1} & \frac{V^y V^r}{V^t+1} & 0 \\ V^r & \frac{V^y V^r}{V^t+1} & 1 + \frac{(V^r)^2}{V^t+1} & 0 \\ 0 & 0 & 0 & \mathbb{I}_7 \end{pmatrix} \tag{19}$$

where \mathbb{I}_7 is a 7×7 identity matrix.

Finally we turn to the tidal forces which the observer experiences as $r \rightarrow 0$. Here, one boosts curvature components R^{klmn} calculated in the stationary frame to the observer's rest frame with $\hat{R}^{abcd} = L^a_k L^b_l L^c_m L^d_n R^{klmn}$. Given that all of the components of R^{klmn} are finite, divergences can only arise through the boost factors. Hence, we consider the behavior of the ten-velocity (18) as $r \rightarrow 0$: $V^t \simeq (\omega - p)\sqrt{B_0 Q} r^{(-\beta-1)/2} \simeq -V^r$ while $V^y \simeq 0$. Hence the observer is accelerated to almost a null radial geodesic as she nears $r = 0$ and L approaches an infinite radial boost. In this limit, one may drop the V^y terms in eq. (19).

With four L 's in the transformation of the curvature, naively the worst divergence would be $O((V^r)^4)$, which would appear in \hat{R}^{trtr} . However, one finds using $(V^t)^2 - (V^r)^2 = 1$ that $\hat{R}^{trtr} = R^{trtr}$. So as a result of a precise cancellation of terms, only a finite contribution remains. Similarly for components where three L 's appear, one finds

$$\hat{R}^{tatr} \simeq |V^r|(R^{tatr} - R^{ratr}) \simeq -\hat{R}^{ratr} \tag{20}$$

where $a \neq r, t$. These components diverge but only as a single power of V^r , and in fact, these singularities would vanish in the special case that $R^{ratr} = R^{tatr}$. The previous divergences are in fact subleading compared to components such as

$$\begin{aligned}\hat{R}^{tatb} &= (V^r)^2(R^{tatb} - R^{tarb} - R^{ratb} + R^{rarb}) + R^{tatb} \\ \hat{R}^{rarb} &= (V^r)^2(R^{tatb} - R^{tarb} - R^{ratb} + R^{rarb}) + R^{rarb} \\ \hat{R}^{tarb} &= -(V^r)^2(R^{tatb} - R^{tarb} - R^{ratb} + R^{rarb}) + R^{tarb}\end{aligned}\tag{21}$$

where $a, b \neq r, t$. So here one finds the naively expected divergence of $O((V^r)^2)$, except for the exceptional circumstance that $R^{tatb} - R^{tarb} - R^{ratb} + R^{rarb} = 0$, in which case these components are invariant. Thus the geometric symmetries of the Riemann tensor dictate that the worst divergence is only $(V^r)^2$.

Implementing the transformation (19) to the curvature components (17), the leading divergences appear in

$$\begin{aligned}\hat{R}^{tyty} &\simeq -\frac{\beta(\beta-1)B_0}{2Q}(\omega-p)^2 r^{(-\beta-1)} \\ \hat{R}^{tatb} &\simeq \delta^{ab} \frac{l(l+1)B_0}{4Q}(\omega-p)^2 r^{(-\beta-1)} \quad \text{for } a, b = \theta, \phi \\ \hat{R}^{titj} &\simeq \delta^{ij} \frac{n_i^2}{2R_i^2} B_0 Q (\omega-p)^2 r^{(-\beta-1)} \quad \text{for } i, j = 5 \dots 9\end{aligned}\tag{22}$$

as well as $\hat{R}^{rarb} \simeq \hat{R}^{tatb} \simeq -\hat{R}^{tarb}$. Hence quite generically the perturbations (14) produce singular tidal forces on the null surface $r = 0$. Because these divergences will not be cancelled by any other terms of the metric for slowly oscillating waves, one may conclude that all of the new solutions have a null singularity at $r = 0$. Hence, the excitation of the wave-like perturbations generically destroys the horizon, producing a naked singularity instead. There is one exception to this conclusion. If $l = 0$ and $n_i = 0$, for which $\beta = 1$, the coefficients of the potentially divergent terms all vanish — this applies to both the quadratic and linear divergences. In this special case, the remaining curvature components (17) are boost invariant. Hence it seems that there is a single family of wavy perturbations which qualifies as black hole hair.

4 DISCUSSION

We have shown that the wave-generating technique of Garfinkle and Vachaspati can be applied to certain higher dimensional black holes. While all scalar curvature invariants are

left unchanged by this construction, generically the new waves produce parallelly propagated curvature singularities. Hence in these cases, the horizon is destroyed and a null singularity is produced instead. Thus it seems that these higher dimensional black holes still respect the spirit of the no-hair theorems. The only nonsingular waves are those with $l = 0 = n_i$. In this case, the original solution (4) is mapped to

$$\begin{aligned}
ds^2 = & -\frac{\left(1 - \frac{Q}{R}\right)^2}{1 + \frac{b(u)}{R}} dt^2 + \frac{dR^2}{\left(1 - \frac{Q}{R}\right)^2} + R^2(d\theta^2 + \sin^2 \theta d\phi^2) \\
& + \left(1 + \frac{b(u)}{R}\right) \left(dy - \frac{Q + b(u)}{R + b(u)} dt\right)^2 + \sum_{i=5}^9 (dx^i)^2
\end{aligned} \tag{23}$$

where the wave profile $b(u)$ has an arbitrary dependence on $u = t - y$. A constant b would yield a shift in the momentum flowing along y . In general, these perturbations represent longitudinal waves carrying momentum in the y direction without transverse oscillations. These nonsingular waves are also distinguished in that they are the only localized waves which drop off slowly enough, *i.e.*, $1/R$, to be detected in the asymptotic region in either the energy or momentum density[7].

While no evidence of a curvature singularity has been found in the present analysis, it may be that more subtle singularities remain to be uncovered — see for example [12]. Normally the approach to proving the existence of a singularity-free horizon would be to find coordinates in which the metric is analytic at the null surface in question. Some progress has been made in finding such coordinates[7]. An essential feature of the examples where such coordinates have been found is that $b(u) \rightarrow \text{constant}$ as $u \rightarrow \pm\infty$. In fact, when the profile does not approach a constant, Horowitz and Yang[13] have found that a mild singularity is produced. In this case, the nonsingular perturbations seem not to represent wavy hair for these black holes, but rather transient waves similar to those which might be produced in a gravitational collapse producing a black hole.

From this point of view, it may seem surprising that the remaining localized waves produced singularities, since one might have expected that a gravitational collapse could also produce such waves. One can investigate the strength of the singularities for $\beta > 1$. Along the geodesics investigated above, one finds that the divergent curvatures $\hat{R} \propto \tau^{-2}$, where τ is the proper time along the curve. Hence these singularities are by no means integrable, and would result in an extended probe being crushed.

As a final comment, we should mention that in ref. [14], string theory was used to produce a statistical mechanical understanding of the black hole entropy for the solution in eq. (4).

In ref. [10], these calculations were extended to include certain wavy excitations, including that in eq. (23). It is interesting that the latter analysis succeeded despite the presence of the singularity later found in ref. [13].

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